Algorithms for Data Science
Data Streams II

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M2 Data Science

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Data Streams

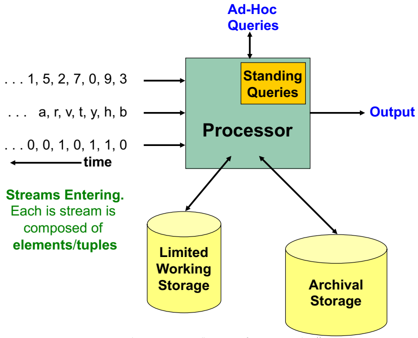
Input rate is controlled externally – so the data processor has no control over the speed of the data

Data streams are:

- infinite – one does not know the size of the data
- non-stationary – the distributions of the data can change (seasonally, daily, hourly)

Model: infinite sequence of items $S = (i_1, i_2, \ldots, i_k \ldots)$

Stream Processing Model



J. Leskovec, A. Rajaraman, J. Ullman: Mining of Massive Datasets, http://www.mmds.org

Objective: asking queries on the stream – *standing* and *ad-hoc*

Restrictions: storage space and processing time – have to process it or we lose it forever!

Implementation Issues

If we had enough memory / time – data streams **would be easy**

With **restrictions**:

- more efficient to get **approximate** answers
- use space-saving techniques such as **hashing**

Problems Studied

- Sample data from a stream
- Filtering items
- Counting distinct elements
- Estimating moments
- Queries over sliding windows

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Count-Distinct Problem

Problem: count the number of distinct items in a data stream

Applications:

- how many different words are in webpages (spam detection)?
- how many distinct products are sold in the last week?
- how many new stars do we find in space?

Count-Distinct Problem

Naïve Approach keep a set of new items found and keep a count of its size

What if we do not have enough space for all the distinct elements?

- we still want an unbiased estimator of the counts
- we accept some error in the estimation as trade-off for space

Flajolet-Martin Approach [?]

Algorithm – assume we have N items in the universe:

1. pick a hash function h mapping the N items to at least $\log_2 N$ bits
2. for each stream item s, $r(s)$ is the number of trailing 0s in the bit representation
   - for instance assume $h(s) = 12$, bit representation **1100**
   - $r(a)$ is then equal to 2
3. keep $R = \max_s r(s)$ over the entire stream

Estimator: the number of distinct items seems thus far is 2^R.

Intuition on Why It Works

Assumption: h hashes with equal probability to all N values, the values from the stream come uniformly

$h(s)$ is a **sequence of $\log_2 N$ bits**:

- a proportion of 2^{-1} (50%) will have $r(s) = 1$
- a proportion of 2^{-2} (25%) will have $r(s) = 2$
- **generally**, a proportion of 2^{-r} will have r trailing 0s

For an **uniform hash function**, it takes thus $1/2^{-r} = 2^r$ items before **we see one with r trailing 0s**

Note: it can be done with trailing 1s, or any other bit function allowing us to compute the probability

Drawbacks and Optimization

Main drawback: the expectation $\mathrm{E}[2^R]$ can get very high

Can fix by using multiple estimators – m different hash functions

- taking the average can overestimate – if one estimator is an outlier
- taking the median is better – but it is always a power of 2
- **best approach**: hybrid, divide the hash functions in groups, compute average in each group, take the median over groups

Space Cost

Minimizes space used

- only have to keep R for each hash function
- we can use as many hash functions as memory permits
- time trade-off: if too many – computing the hashes (and maintaining averages, medians) can be too time costly

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Moments of a Sequence

Assume we have a sequence/stream S having N possible distinct (ordered) values, and m_i is the number of times the ith distinct element appears in S

Moment: the nth moment of a sequence S is equal to

$$\sum_{i \in S} (m_i)^n.$$

Moments of a Sequence

Example of moments:

1. 0th moment: the number of distinct items in the stream – can be estimated using the approach presented before!
2. 1st moment: the length of the stream – easy to keep count of
3. 2nd moment: surprise number – how uneven the distribution is

Challenge: same as distinct items in stream – cannot keep all values in memory

Surprise Number

5 distinct elements not varying much: `5 4 4 4 3`

- 2nd moment (surprise number): $1^2 + 3^2 + 1^2 = 11$

5 distinct elements with outliers: `16 1 1 1 1`

- 2nd moment (surprise number): $1^2 + 4^2 = 17$

Alon-Matias-Szegedy Algorithm [?]

Assume a stream has a length n, and we have space to store a few variables and not all m_i

We keep some variables X:

- X.val – the value of the element
- X.c – the count of that element in the stream

Alon-Matias-Szegedy Algorithm (AMS):

1. choose a number i between 1 and n
2. when the stream S reaches i, set $X.\text{val} = s_i$ and $X.\text{c} = 1$
3. everytime the value in X.val is encountered in S, increment X.c

Using AMS for Estimating 2nd Moment

Estimate of the 2nd moment is:

$$n(2X.\mathrm{c} - 1)$$

The estimate can be refined by using k different X variables; the estimate is then the average of the estimates:

$$\frac{n}{k} \sum_{i \in \{1,\ldots,k\}} (2X_i.\mathrm{c} - 1)$$

AMS example

Stream ($n = 15$):

```
a b c b d a c d a b d c a a b
```

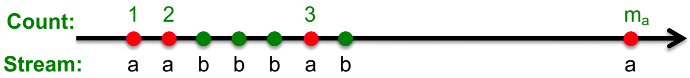
- surprise number $5^2 + 4^2 + 3^2 + 3^2 = 59$

Keep X_1, X_2, X_3, and choose $3, 8, 13$ as random positions in the stream:

- $X_1.\text{val} = c$, and – at the end of the stream – $X_1.\text{c} = 3$
- $X_2.\text{val} = d$, and – at the end of the stream – $X_2.\text{c} = 2$
- $X_3.\text{val} = a$, and – at the end of the stream – $X_3.\text{c} = 2$

The final estimate is:
$15/3 \times ((2 \times 3 - 1) + (2 \times 2 - 1) + (2 \times 2 - 1)) = 55$

Why It Works



Let us write $f(X) = n(2c - 1)$, and c_t the number of times an item appears from time t on

We need to give a bound on the **expectation** of f:

$$\mathrm{E}[f(X)] = \frac{1}{n}\sum_{t=1}^{n} n(2c_t - 1) = \sum_{i=1}^{m_i}(2i - 1)$$
$$= 2\frac{m_i(m_i+1)}{2} - m_i = (m_i)^2$$

– **in expectation**, the formula is exactly the **second moment**!

Estimating Higher Order Moments

The algorithm works for **any moment k**, but the estimate changes

General estimator

$$n\left(c^k - (c-1)^k\right)$$

Infinite Streams

What happends when we do not know n?

- assume we can only hold k functions

We can use **Reservoir Sampling**

- choose the first k times for k variables
- for $n > k$ choose the item as a new variable with probability k/n, if chosen discard one of the previous k randomly
- in the estimator, use the current length of the stream as n

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Setting: sometimes we only need to query the last N elements of a stream – queries over a sliding window

- N can be very large
- there can also be multiple stream, so keeping multiple windows is too much

Example: transactions (product was sold, ad was clicked, etc.)

Sliding Windows

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

q w e r t y u i o p a s d f g h j k l z x c v b n m

← Past Future →

J. Leskovec, A. Rajaraman, J. Ullman: Mining of Massive Datasets, http://www.mmds.org

Counting Bits

Problem: given a stream S of 0 and 1, we want to answer queries of the form

- how many 1s are in the last k bits ($k \leqslant N$)

Assumption: we cannot afford to keep the most recent N bits

- but, impossible to get an exact answer without storing the entire window
- have to settle for approximate answers

Non-Uniform Streams

In uniform streams, we can simply estimate the number of 1s by counting the number of 1s as a, 0s as b and estimate as

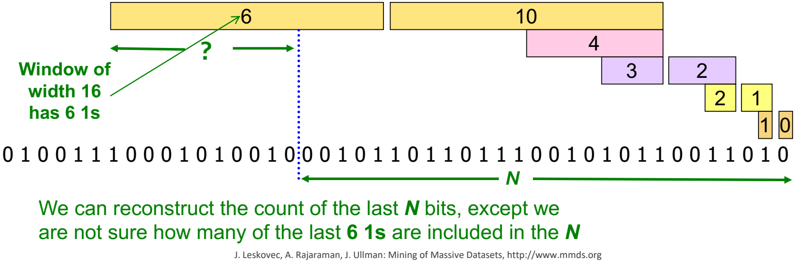
$$N\frac{a}{a+b}$$

But streams are not uniform!

Datar-Gionis-Indyk-Motwani (DGIM) Method [?]

Main Idea – exponential windows

- summarize regions of the streams in buckets, that are exponentially increasing
- keep the count for each



Pros and Cons

The advantages:

- only needs $\mathcal{O}(\log^2 N)$ bits – $\mathcal{O}(\log N)$ counts of $\log_2 N$ bits
- easy updates
- error in count not greater than the number of 1 in the “last” area
- if 1s are (relatively) evenly distributed, error is no more than 50%

The big disadvantage:

- if all the 1s are in the unknown area – error is unbounded!

The DGIM Fix

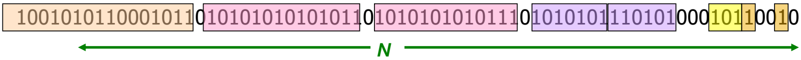
Main idea: instead of keeping fixed sizes of buckets, keep buckets containing a fixed size of **1**s

- the windows increase exponentially – numbers of **1** kept as powers of **2**, e.g., `1 1 2 4 16`

Buckets contain:

- the timestamp of its end – kept as timestamp modulo N, needs $\mathcal{O}(\log N)$ bits
- the number of **1**s in it – since powers of **2** always, it only needs $\mathcal{O}(\log \log N)$

Restrictions on Buckets



- at most one or two buckets of the same size
- no overlap of timestamps
- new buckets are smaller than earlier ones
- buckets are removed when end time $> N$

Updating Buckets

When a new item (bit) comes, drop the last bucket if end-time after N

Update depends on the bit (0 or 1):

1. if bit is 0 – no changes needed
2. if bit is 1:
   - create a new bucket of size 1
   - if 3 buckets of size 1, combine oldest two in a new bucket of size 2
   - recurse on sizes

Updating Buckets

Current state of the stream:

1001010110001011010101010101010110101010101110101010111010100010110010

Bit of value 1 arrives

0010101100010110101010101010101101010101011101010101110101000101100101

Two orange buckets get merged into a yellow bucket

0010101100010110101010101010101101010101011101010101110101000101100101

Next bit 1 arrives, new orange bucket is created, then 0 comes, then 1:

0101100010110101010101010101101010101011101010101110101000101100101101

Buckets get merged…

0101100010110101010101010101101010101011101010101110101000101100101101

State of the buckets after merging

0101100010110101010101010101101010101011101010101110101000101100101101

J. Leskovec, A. Rajaraman, J. Ullman: Mining of Massive Datasets, http://www.mmds.org

Querying

Query:

1. sum the sizes of all buckets except the last
2. add half the size of the last (we do not know the proportion of the last window in N)

Error is at most 50%:

- can be reduced by maintaining r or $r - 1$ buckets of each size
- error is then at most $\mathcal{O}(1/r)$
- trade-off between number of bits and the error

Extensions

Using $k < N$ as a query parameter:

- want to query only the last k bits in the window N
- can simply "cut" at k and use the same estimator

Sum of last k integer elements:

- assume integers have at most m bits
- treat each bit as a separate stream and count the 1 in last k
- estimate as $\sum_{i=0}^{m-1} c_i 2^i$ where c_i is the DGIM estimator for bit i

Acknowledgments

The contents follows Chapter 4 of [**?**]. Figures in slides 4, 21, 26, 29, 32, and 34 are taken from `https://www.mmds.org/`

References i